Exercise Sheet 1: Solutions

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1. (a) $E(X+Y) = E(X) + E(Y) = 4 + 3 = 7.$

   (b)

$$\begin{aligned} Var(X+Y) &= Var(X) + Var(Y) + 2Cov(X,Y) \\ &= 2 + 1 + 2 \times 0.5 = 4. \end{aligned}$$

   (c) $E(3X) = 3E(X) = 3 \times 4 = 12.$

   (d) $Var(3X) = 3^2 Var(X) = 9 \times 2 = 18.$

2. (a)

$$\begin{aligned} E(A) &= \sum_{i=1}^{4} A_i P(A_i) \\ &= 1 \times 0.3 + 2 \times 0.1 + 3 \times 0.4 + 4 \times 0.2 = 2.5. \end{aligned}$$

   (b)

$$\begin{aligned} E(A^2) &= \sum_{i=1}^{4} A_i^2 P(A_i) \\ &= 1 \times 0.3 + 4 \times 0.1 + 9 \times 0.4 + 16 \times 0.2 = 7.5. \end{aligned}$$

   (c)

$$\begin{aligned} Var(A) &= \sum_{i=1}^{4} (A_i - E(A))^2 P(A_i) \\ &= (-1.5)^2 \times 0.3 + (-0.5)^2 \times 0.1 + (0.5)^2 \times 0.4 + (1.5)^2 \times 0.2 \\ &= 1.25 \end{aligned}$$

   or, alternatively,

$$\begin{aligned} Var(A) &= E(A^2) - E(A)^2 \\ &= 7.5 - (2.5)^2 = 1.25. \end{aligned}$$

3. (a)

$$\sum_{i=1}^{3} B_i = 10 + 50 + 30 = 90.$$

(b)

$$\sum_{i=1}^{3} B_i^2 = 100 + 2500 + 900 = 3500.$$

(c)

$$\begin{aligned} \sum_{j \neq i} \sum_{i=1}^{3} B_i B_j &= 10 \times 50 + 10 \times 30 + 50 \times 10 \\ &\quad +50 \times 30 + 30 \times 10 + 30 \times 50 \\ &= 4600. \end{aligned}$$